Combinatorics and Graph Theory

M. Math. II

Semestral Examination

Instructions: All questions carry ten marks. All graphs are assumed to be simple. Results proved in the course can be used without proof.

1. Prove that the number of points in a Steiner triple system must be 1 or 3 modulo 6.

2. Prove that Hall's matching theorem for bipartite graphs is equivalent to the König's theorem on maximum matching in a bipartite graph.

3. Prove that every planar graph with at least four vertices have at least four vertices of degree less than 6.

4. Compute the characteristic polynomial of the following graphs

   (a) The complete graph K_n

   (b) The complete bipartite graph $K_{n,m}$

5. Let (X_1, Y_1) and (X_2, Y_2) be minimum cuts in a transportation network. Show that $(X_1 \cup X_2, Y_1 \cap Y_2)$ is also a minimum cut.

6. Prove that the MaxFlow-MinCut theorem implies the Hall's matching theorem for bipartite graphs.